Mathematics Department
Topics for Qualification Exam

PhD Level Candidates
2023-2024

1. Graph Theory

Abstract Algebra

Basics

0.1 Sets

0.2 Introduction to Groups (Basic Axioms and Examples)

- Binary operation (Definition, Properties)
- Mathematical system (Definition, Examples)
- Semi-group (Definition, Examples)

Groups

1.1 Group (Definition, Properties, Examples)

1.2 Group of integers modulo n

1.2.1 Congruent modulo n (Definition, Properties)

1.3 Cyclic group (Definition, Properties)

1.4 Finite group (Definition, Examples)

1.5 Permutation mapping (Definition, Examples)

Subgroups

2.1 Subgroup (Definition, Properties, Examples)

2.2 Center of a group (Definition, Properties)

2.3 Order of an element of a group

2.4 Groups of cosets (Definition, Properties)

2.5 Index of a subgroup

2.6 Lagrange Theorem

2.7 Non-trivial subgroup

Normal Subgroups and Quotient Groups

References

[1] McCoy N. H. and Berger T. R., Algebra: Groups, Rings and Other Topics, Allyn and Bacon, Inc. Boston London Sydney Toronto, 1977.
[2] Fraleigh J. B., A first Course in Abstract Algebra, ADDISON-WESLEY PUBLISHING COMPANY, 1982.
[3] Herstei I. N., Topics in Algebra, JOHN WILEY & SONS New York Chichester Brisbane Toronto Singapore, 1975.
[4] Allenby R B J T, Rings, Fields and Groups, Edward Arnold, 1983.
Dummit D. S. and Foote R. M., Abstrat Algebra, John Wiley & Sons, Inc., 2003.
[5] Singh S. and Zameeruddin Q., Modern Algebra, VIKAS PUBLISHING HOUSE PVT LTD, 1972.
[6] Durbin J. R., Modern Algebra, JOHN WILEY & SONS
New York Chichester Brisbane Toronto Singapore, 1985.
[7] Gallian J. A., Contemporary Abstract Algebra, HOUGHTON MIFFLIN COMPANY, 1998.

Graph Theory

1. Graphs and subgraphs.
2. Connected and disconnected Graphs.
3. Trees.
4. Planarity.
5. Connectivity and Networks.

6. Hamiltonian and Eulerian graphs.
7. Graph Coloring.
8. Digraphs.

References:

[1] Chartrand, G. and Lesniak, L., Graphs and Digraphs, 2nd Edition, Wadsworth and Books/Cole Advanced Books & Software Pacific cover, Californiya, 1986.
[2] Harary, F., Graph Theory, Addison-Wesley Publishing Company, 1968.

Advanced Calculus

1. Infinite Sequence and series
- a. convergent sequence and series
- b. Testing series for convergence
- c. power series
- d. Taylor Series

2. Polar coordinates
- a. Graphing Polar coordinates
- b. Area of Polar Curves
- c. Arc Length in Polar coordinates

3. Partial Derivatives
- a. Functions of Several Variables
- b. Limits and continuity in higher dimensions
- c. Partial Derivatives
- d. The Chain rule
- e. Directional derivatives and gradient vectors
- f. Tangent planes and differentials
- g. Extreme values and saddle points
- h. Lagrange multipliers

4. Vector valued functions and motion in space
- a. Vectors, Dot and Cross Products
- b. Orthogonal and perpendicular vectors
- c. Equations of Lines and Planes
- d. Functions and Surfaces
- e. Cylindrical and Spherical Coordinates
- f. Vector Functions
- g. Arc Length, Curvature and Motion in Space
- h. Surfaces Defined by Parametric Equations
- i. Vector identities, mapping
- j. Gradient, divergence and curl
- k. Line, surface and volume integrals

5. Multiple Integrals
- a. Double and Iterated Integrals
- b. Triple Integrals
- c. Lagrange Multiplier

d. Spherical, Cylindrical coordinates
e. Change of Variables
f. Green’s Theorem Useful

References:

1. Anton, H. Calculus with Analytic Geometry, 5th ed., John Wiley & Sons, New York, N.Y. 1995.
2. Grossman S. I. Calculus, University of Montana. U.S.A, 1977

Mathematical Analysis

Ch-I: The Real Number System

1. Introduction and Inadequacy of the Rationales.
2. Upper and Lower Bounds.
3. Maximum and Minimum.
4. Supermum and Infimum.
5. Archimedean Ordered Fields.
6. Completeness Axiom.

Ch-II: Metric Spaces

1. Some Necessary Inequalities.
2. Basic Metric Notions in Rn.
3. General Metric Space (different type of metric spaces).
4. Some Topological Concepts. (Openness, Closeness, Limit points, Closures).
5. Union and intersection of closed and opened sets.
6. Metric Subsequences.

Ch-III: Completeness and Compactness in Metric Spaces

1. Cauchy Completion of Real Number System.
2. Theorems on Completeness.
3. Compactness of Metric Spaces.
4. Hein Borel Theorem
5. Relation between Completeness and Compactness

Ch-IV: Limits and Continuity in the Metric Spaces

1. Definition of the Limit of the Functions.
2. Equivalent Definition of the Limit.

3. Continuity at a Point.
4. Continuous Functions on Compact Sets.
5. Uniform Continuity.
6. Discontinuities Points and their Types.
7. Compactness and Uniform Continuous.

Ch-V: Sequence and Series of Functions in Metric Spaces

1. Sequence of Functions.
2.. Pointwise Convergent and Uniform Convergent.
2. Power Series and Series of Functions.
3. Uniform Convergence and Continuity.
4. Subsequences.

Ch-VI: Differentiation

1. A New Concept of Differentiation (Definitions and Examples).
2. Properties of Differentiation.
3. Differentiation and Continuity.
4. Intermediate Value Property.
5. Rolle's Theorem, Lagrange Mean Value Theorem, Cauchy Mean Value Theorem
6. Smooth Functions and Taylor Theorem.
7. Analytic Functions.

Ch-VII: Riemann Integral

1. Riemann Partition.
2. Riemann Sum (Upper and Lower Riemann Sum).
3. Riemann Integral (Upper and Lower Riemann Integral).
4. Continuity and Integration.
5. Some Necessarz Examples.
6. Zero Sets.
7. Cantor Set (optional)
8. Lebesgue Theorem for Riemann Integral.
9. Integrability of a Sequence of Functions and its Limit.
10. Integration and Inverse Derivative.
11. Differentiation and Riemann Integration. (Fundamental Theorems)

Ch-IX: Elementary Measure Theory

1. Length of Bounded Open Intervals.
2. Length of Bounded open Sets.
3. Length of Bounded Sets.
4. Measure of Bounded Sets (Inner and Outer Measure).
5. Basic Theoretical Properties of the Measure of Sets.
6. Measure of Unbounded Sets.

7. Non-measurable Sets. (optional)
8. Measurable Functions (Definition and Elementary Properties). (optional)

Ch-X: Lebesuge Integral

1. Lebesuge Partition.
2. Lebesuge Lower and Upper sum
3. Lebesuge Lower and Upper Integral.
4. Integrability and Measurability (Basic Theoretical Properties and Examples).
5. Lebesuge Integral for Bounded Functions.
6. Lebesuge Integral for Unbounded Functions. (optional)

References

[1] Apostol T. M., Mathematical Analysis.2ed, Addison Wesley Pub. Comp., 1978
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[3] Brgant V., Metric Spaces.Cambridge Univ. Press, 1985
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Linear Algebra

Contents:
1. Vector Spaces and Subspaces.
2. Linear Transformations.
3. Matrices of linear Transformations.
4. Eigenvectors and Eigenvalues.
5. Diagonalization.
6. Orthogonality.

References:

[1] Kolman,_B. and Hill, D., Elementary Linear Algebra, 9th edition, Pearson Education, Inc, 2008.
[2] Larson, R., Elementary Linear Algebra, 7th Edition, Brooks/Cole, Cengage Learning, 2013.

2. Numerical Analysis

Numerical Analysis

Contents:

References

Burden, R. L. and Faires, J. D. (2011), Numerical Analysis, 9th edition, BROOKS/ COLE.
Or any other book that contains these topics

Partial Differential Equations

Contents:

4.2 Derivation of One-dimensional Wave Equation.

4.3 Solution of One-dimensional Wave Equation by Canonical Reduction.

4.4 Periodic Solution of One-dimensional Wave Equation in Cylindrical Coordinates

4.5 Periodic Solution of One-dimensional Wave Equation in Spherical Polar.

References:
A. D. Polyanin, Handbook of Linear Partial Differential Equations for Engineers and Scientist, Chapman & Hall/CRC, (2002).
K. S. Rao, Introduction to PARTIAL DIFFERENTIAL EQUATIONS, THIRD EDITION, PHI Learning Private Limited, New Delhi., (2011)
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3. Ordinary Differential Equations (ODEs)

Bifurcation Theory

Bifurcation Theory

- Types of bifurcation theory.
- Structural Stability and Pcixoto's Theorem.
- Local one-dimensional bifurcations (saddle-node, fold, transcritical and pitchfork bifurcations).
- Hopf and zero Hopf bifurcations.
- Local bifurcations of nonlinear Systems in the plane.
- Local bifurcations at nonhyperbolic equilibrium points in three dimensions.
- Local bifurcations at nonhyperbolic periodic orbits.

References:

[1] *Arrowsmith, D. and Place, C.M., 1992. Dynamical systems: differential equations, maps, and chaotic behaviour (Vol. 5). CRC Press.*

[2] *Lynch, S., 2009. Dynamical systems with applications using MapleTM. Springer Science & Business Media.*

[3] *L. Perko, Differential Equations and Dynamical Systems, Springer,New York.: Texts in Applied Mathematics, 2006*

Qualitative Theory of Differential Equations

Qualitative Theory of Differential Equations

- Existence and Uniqueness of solutions of differential equations
- Existence Theorem: The method of successive approximation
- Autonomous equations
- Dynamical systems in plane
- Linear plane autonomous systems
- Simple Canonical systems
- Non-simple Canonical systems
- Classification of simple linear phase portrait in the plane
- Non-linear System in the plane
- The linearization Theorem
- First integrals
- Limit Cycles

References:

1. D. K. Arrowsmith,C. M. Place. Ordinary differential equations: a qualitative approach with applications. Chapman and Hall, 1982.
2. Otto Plaat. Ordinary differential equations, 1971.

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[2] Larson, R., Elementary Linear Algebra, 7th Edition, Brooks/Cole, Cengage Learning, 2013.

4. Statistics

Advanced Mathematical Statistics (Statistical Inference)

Advanced Mathematical Statistics (Statistical Inference)

- Distributions Functions of Discrete and Continuous Random Variables;
- Special Distribution Functions;
- Sampling Theory; Population, Sample, Parameter and Statistic;
- Estimation of Parameters and Methods of Estimation: Moment Method; Maximum Likelihood Method and Criteria for Good Estimators:
  Minimum Variance Unbiased Estimation;
  Cramer-Rao Lower Bound;
- Interval Estimation;
  Confidence Interval for: Mean, Variance, Difference between Means, Ratio for Variances
- Hypothesis Tests: Null and Alternative Hypothesis;
- Sample Tests: Z-Test; T-Test; F-Test and Chi-Square Test;
- Types of Errors; Type I and Type II Error; P-Value.

References:

- Hogg, R., McKean, J. and Graig, A. (2019), Introduction to Mathematical Statistics, Pearson Education, Inc. 8th edition, New York
- J. K. WANI, (1971), Probability and Statistical Inference, Meredith Corporation, 440 Park Avenus south, New York.

- Dennis D. W., William M. III, Richard L. S. (2008), Mathematical Statistics with Applications, Tomson Learning, USA.

Statistical Data Analysis

Statistical Data Analysis Course

1. The Role of Statistics and the Data Analysis Process

1.1 Reasons to Study Statistics;
1.2 The Nature and Role of Variability;
1.3 Statistics and the Data Analysis Process;
1.4 Types of Data and Some Simple Graphical Displays;
1.5 Collecting Data;
1.1 Observation and Experimentation;
1.2 Sampling.

2. Graphical Methods for Describing Data

2.1 Displaying Categorical Data;
2.2 Comparative Bar Charts and Pie Charts;
2.3 Displaying Numerical Data;
2.4 Frequency Distributions and Histograms; Displaying Bivariate Numerical Data;
2.5 Interpreting and Communicating the Results of Statistical Analyses.

3. Numerical Methods for Describing Data

3.1 Describing the Center of a Data Set;
3.2 Describing Variability in a Data Set;
3.3 Summarizing a Data Set: Boxplots;
3.4 Interpreting Center and Variability.

4. Summarizing Bivariate Data

4.1 Correlation;
4.2 Linear Regression;
4.3 Fitting a Line to Bivariate Data;
4.4 Assessing the Fit of a Line;
4.5 Nonlinear Relationships and Transformations;

4.6 Logistic Regression;
4.7 Interpreting and Communicating the Results of Statistical Analyses.

5. Random Variables and Probability Distributions

5.1 Random Variables;
5.2 Probability Distributions for Discrete and continuous Random Variables;
5.3 Probability Distributions for Discrete and Continuous Random Variables;
5.4 Binomial, Geometric, Normal Distributions;
5.5 Checking for Normality and Normalizing Transformations.

6. Sampling

6.1 Variability and Sampling Distributions;
6.2 Statistics and Sampling Variability;
6.3 The Sampling Distribution of a Sample Mean;
6.4 The Sampling Distribution of a Sample Proportion;
6.5 Estimation and point estimation;
6.6 Large-Sample Confidence Interval for a Population mean;
6.7 Confidence Interval for variance;
6.8 Confidence Interval for Proportion.

7. Testing Hypothesis

7.1 Hypothesis Testing Using a Single Sample;
7.2 Hypotheses and Test Procedures;
7.3 Errors in Hypotheses Testing;
7.4 Large-Sample Hypothesis Tests for a Population Proportion;
7.5 Hypotheses Tests for a Population Mean;
7.6 Power and Probability of Type II Error

8. The Analysis of Categorical Data and Goodness-ofFit; Tests and Regression

8.1 Chi-Square Tests for Univariate Data;
8.2 Tests for Homogeneity and Independence in a Two way;
8.3 Simple Linear Regression Model;
8.4 Checking Model Adequacy;
8.5 Inferences Based on the Estimated Regression Line Population Correlation Coefficient;
8.6 Multiple Regression Models and fitting a Model and Assessing Its Utility;

8.7 Inferences Based on an Estimated Model;
8.8 Single-Factor ANOVA and the F Test;
8.9 Multiple Comparisons- The F Test for a Randomized Block ;
8.10 Two-Factor ANOVA.;
8.11 Distribution-Free Procedures for Inferences About a Difference Between Two Population or Treatment Means Using Independent Samples ;
8.12 Distribution-Free ANOVA.

References:

Peck, R., Olsen, C., Devore, J. (2008) "*Introduction to Statistics and Data Analysis*", Thomson Higher Education, Third Edition.USA.

Taylor, J. (2020), "*Statistical Techniques for Data Analysis*", Chapman & Hall.

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3. General Metric Space (different type of metric spaces).
4. Some Topological Concepts. (Openness, Closeness, Limit points, Closures).
5. Union and intersection of closed and opened sets.
6. Metric Subsequences.

Ch-III: Completeness and Compactness in Metric Spaces

1. Cauchy Completion of Real Number System.
2. Theorems on Completeness.
3. Compactness of Metric Spaces.
4. Hein Borel Theorem
5. Relation between Completeness and Compactness

Ch-IV: Limits and Continuity in the Metric Spaces

1. Definition of the Limit of the Functions.
2. Equivalent Definition of the Limit.
3. Continuity at a Point.
4. Continuous Functions on Compact Sets.
5. Uniform Continuity.
6. Discontinuities Points and their Types.
7. Compactness and Uniform Continuous.

Ch-V: Sequence and Series of Functions in Metric Spaces

1. Sequence of Functions.

2.. Pointwise Convergent and Uniform Convergent.

2. Power Series and Series of Functions.
3. Uniform Convergence and Continuity.
4. Subsequences.

Ch-VI: Differentiation

1. A New Concept of Differentiation (Definitions and Examples).
2. Properties of Differentiation.
3. Differentiation and Continuity.
4. Intermediate Value Property.
5. Rolle's Theorem, Lagrange Mean Value Theorem, Cauchy Mean Value Theorem
6. Smooth Functions and Taylor Theorem.
7. Analytic Functions.

Ch-VII: Riemann Integral

1. Riemann Partition.
2. Riemann Sum (Upper and Lower Riemann Sum).
3. Riemann Integral (Upper and Lower Riemann Integral).
4. Continuity and Integration.
5. Some Necessarz Examples.
6. Zero Sets.
7. Cantor Set (optional)
8. Lebesgue Theorem for Riemann Integral.
9. Integrability of a Sequence of Functions and its Limit.

10. Integration and Inverse Derivative.
11. Differentiation and Riemann Integration. (Fundamental Theorems)

Ch-IX: Elementary Measure Theory

1. Length of Bounded Open Intervals.
2. Length of Bounded open Sets.
3. Length of Bounded Sets.
4. Measure of Bounded Sets (Inner and Outer Measure).
5. Basic Theoretical Properties of the Measure of Sets.
6. Measure of Unbounded Sets.
7. Non-measurable Sets. (optional)
8. Measurable Functions (Definition and Elementary Properties). (optional)

Ch-X: Lebesuge Integral

1. Lebesuge Partition.
2. Lebesuge Lower and Upper sum
3. Lebesuge Lower and Upper Integral.
4. Integrability and Measurability (Basic Theoretical Properties and Examples).
5. Lebesuge Integral for Bounded Functions.
6. Lebesuge Integral for Unbounded Functions. (optional)

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Linear Algebra

Contents:
1. Vector Spaces and Subspaces.
2. Linear Transformations.
3. Matrices of linear Transformations.
4. Eigenvectors and Eigenvalues.
5. Diagonalization.
6. Orthogonality.

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